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New theoretical approach to dynamics of heat-mass-transfer, thermal turbulence and air ventilation in atmosphere of an industrial city II. Spectrum of thermal turbulence

In this paper we go on a development of consistent theoretical approach to modelling the turbulent regime in the atmosphere of the industrial cities and present the analytical foundations of a new model of thermal turbulence spectrum for atmosphere of an industrial city. Special attention is paid to general analytical aspects for accounting of the phenomenon of wave or vortex diffusion, which is usually ignored in most works on atmospheric ventilation modelling. Redistribution of energy over the spectrum of eddy sizes is usually called a spectral transformation, the study of which is possible only under the condition of real introduction of nonlinearity into the equation of turbulent motion. The approach presented is implemented into the general theory of heat-mass-transfer, thermal turbulence and air ventilation in atmosphere of an industrial city, including an improved theory of atmospheric circulation in combination with the hydrodynamic modelling, method of a complex geophysical plane field and the Arakawa-Schubert approach to a quantitative description of convective instability in the city's atmosphere.

***Key words**: physics of industrial city's atmosphere, heat-mass-transfer, thermal turbulence, air ventilation in atmosphere*, *vortex diffusion.*

Introduction. One of the most important problems of the modern physics of aerodispersed systems, atmospheric and climate systems, physics of atmosphere of the urban systems and industrial cities is study of an energy-, heat-, mass-transfer in atural continuous environments (e.g.[1-8]). Practically all known modern, as a rule, simplified, approaches allow to estimate the temporal and spatial structure of air ventilation in an atmosphere, a transfer of harmful substances in an atmosphere of the industrial cities significantly and use as the simple molecular diffusion models as system of regression equations (e.g. [7-20]). Disadvantages of these approaches are well known and became very critical if, for example, the atmosphere contains elements of convective instability.

In our previous papers [21-26] we develop the theoretical foundations of a new energy, angle momentum and entropy balance approach to modelling climate and macroturbulent atmospheric dynamics, heat and mass transfer at macroscale as well as its partial theoretical approach to dynamics of heat-mass-transfer, thermal turbulence and air ventilation in atmosphere of an industrial city. The latter includes an advanced theory of atmospheric circulation in combination with the hydrodynamic prediction model (with quantitatively correct account of turbulence in the atmosphere at local scales) and the Arakawa-Schubert model of cloud convection as well as new

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theoretical approach to dynamics of heat-mass-transfer, thermal turbulence (as in a heat island zone as in a city's periphery) and air ventilation in atmosphere of an industrial city.

In this paper we go on a development of consistent adequate approach to modelling the turbulent regime in the atmosphere of the industrial cities and present the key elements of a new model of thermal turbulence spectrum of an industrial city. Special attention is paid to general analytical aspects for accounting of the phenomenon of wave or vortex diffusion, which is usually ignored in most works on atmospheric ventilation modelling. Redistribution of energy over the spectrum of eddy sizes is usually called a spectral transformation, the study of which is possible only under the condition of real introduction of nonlinearity into the equation of turbulent motion. All above said determines the construction of a macro- and meso-meteorological theoretical foundations of a fundamentally new "Green City" technology, which is associated with the development of a complex of new nonlinear-stochastic hydrodynamic models for the quantitative description of the dynamics of atmospheric ventilation of large industrial cities, taking into account meteorological, anthropogenic, orographic and other factors, a new generalized approach to the analysis and modeling of anthropogenic pollution of the atmosphere of industrial cities (which is based on the optimized theory of atmospheric ventilation in an industrial city in combination with a hydrodynamic forecast model with quantitative consideration of turbulence in the atmosphere of the urban area, methods of the complex geophysical field theory and the Arakawa-Schubert approach to the quantitative description of convective instability applied to the modeling of heat-mass transfer and air ventilation in the atmosphere of an industrial city (e.g. [5, 7, 8, 18-25]).

A new approach to modelling the turbulent regime in the atmosphere of industrial places. In order to make modelling a turbulent regime in atmosphere of a industrial city (e.g. [7, 8, 21, 22]), an adequate model should be presented to predict coupling moments, which is described by the Reynolds system of variables, which introduces the concept of the average and fluctuation flow, and itself:

$$u = \overline{u} + u',\ v = \overline{v} + v',\ \omega = \overline{\omega} + \omega',\ \Phi = \overline{\Phi} + \Phi',\ \theta = \overline{\theta} + \theta', \quad (1)$$

where m as usually, Φ is a pressure, θ is a potential temperature; *u*, *v*, ware the velocity components. Then the Reynolds equations are written in the standard form:

$$\frac{\partial \overline{u}_j}{\partial t} + \frac{\partial}{\partial x_k}\left(\overline{u}_k \overline{u}_j + \overline{u'_k u'_j}\right) = -\frac{\partial \overline{p}}{\partial x_j} - \delta_{j3}\frac{g\overline{\theta}}{\theta_0} \quad (2)$$

$$u_1 = u,\ u_2 = v,\ u_3 = \omega,\ \delta_{ij} = \begin{cases} 0 & npu \quad i \neq j \\ 1 & \quad\quad\ i = j \end{cases},$$

And if the index in the monomial expression is repeated twice, it means subsummation from 1 to 3. Further it is natual to add the the standard thermodynamics equation:

$$\frac{\partial \overline{\theta}}{\partial t} + \frac{\partial}{\partial x_k}\left(\overline{u}_k \overline{\theta} + \overline{u'_k \theta'}\right) = 0. \quad (3)$$

Usually, the Reynolds stresses in turbulent motion are parameterized as follows:

$$\frac{\partial}{\partial x_k}\left(\overline{u'_k u'_j}\right) = k\Delta\overline{u}_j \,;\; \frac{\partial}{\partial x_k}\left(\overline{u'_k \theta'}\right) = k\Delta\overline{\theta} \tag{4}$$

where k is a turbulence coefficient, which differs significantly in size for turbulent horizontal vortices, horizontally vertical and purely vertical vortices. The usual atmospheric parameterization with the turbulence coefficient with a very large degree of approximation is used in models of the surface layer, where the concept of isotropy of the vortex motion in all three directions of space is accepted. But in our case of a turbulent atmosphere of an industrial city where turbulent eddies in the horizontal direction differ little in scale from vertical ones, such an approximation is absolutely unacceptable. Therefore, it is necessary to apply equations for predicting the Reynolds stresses, which will become the basis of the closure model for nonlinear processes [7, 8]. The derivation of these equations is carried out on the basis of equations (2) according to the following rule:

$$\frac{\partial u'_j}{\partial t} + \frac{\partial}{\partial x_k}\left(\overline{u}_j u'_k + \overline{u}_k u'_k + u'_j u'_k - \overline{u'_j u'_k}\right) = \frac{\partial p'}{\partial x_j} - \sigma_{j3}\frac{g\theta'}{\theta_0}$$

$$\frac{\partial \theta'}{\partial t} + \frac{\partial}{\partial x_k}\left(\overline{\theta} u'_k + \overline{u}_k \theta' + u'_k \theta' - \overline{u'_k \theta'}\right) = 0\,. \tag{5}$$

The system of closing equations can be written in the following form:

$$\frac{\partial \overline{u'_i u'_j}}{\partial t} + \frac{\partial}{\partial x_k}\left(\overline{u}_k \overline{u}'_i \overline{u}'_j + \overline{u}'_k \overline{u}'_i \overline{u}'_j\right) = \frac{\partial \overline{p' u'_i}}{\partial x_j} + \frac{\partial \overline{p' u'_j}}{\partial x_i} =$$

$$-\overline{u}'_i \overline{u}'_k \frac{\partial \overline{u}'_j}{\partial x_k} - \overline{u'_j u'_k}\frac{\partial \overline{u}'_i}{\partial x_k} - \frac{g}{\theta_0}\left(\delta_{i3}\overline{u'_j \theta'} + \delta_{j3}\overline{u'_i \theta'}\right) + \Phi'\left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right);$$

$$\frac{\partial \overline{u'_i \theta'}}{\partial t} + \frac{\partial}{\partial x_k}\left(\overline{u}_k \cdot \overline{u'_i \theta'} + \overline{u'_k u'_i \theta'}\right) + \frac{\partial \overline{p' \theta'}}{\partial x_i} = \Phi' \frac{\partial \overline{\theta}'}{\partial x_i} - \overline{u'_i u'_k}\frac{\partial \overline{\theta}}{\partial k} - \overline{\theta}' \overline{u}'_k \frac{\partial \overline{u}_j}{\partial x_k} - \delta_{i3}\frac{g}{\theta_0}\theta'^2;$$

$$\frac{\partial \overline{\theta'^2}}{\partial t} + \frac{\partial}{\partial x_k}\left(\overline{u}_k \cdot \overline{\theta'^2} + \overline{u'_k \theta'^2}\right) = -2\overline{u'_k \theta'}\frac{\partial \overline{\theta}}{\partial x_k}\,. \tag{6}$$

As a result, we have 16 equations regarding the Reynolds stress and moments of connection of velocity pulsations with entropy pulsations, since

$$dS = c_p \cdot d \cdot \ln\theta\,, \tag{7}$$

where S is entropy, c_p is the specific heat capacity of the isobaric process.

Then $b^2 = \overline{u'_k u'_k}$ is the kinetic energy of fluctuation; θ'^2 is a measure of process activity, which is directly related to the entropy dispersion S; $\overline{u'_i \theta'}$ is a measure of the relationship between dynamic deformations and the activity of the process.

The unknown quantities in the system of equations (6) can be combined into the so-called 4-tensor [7, 8]:

$$\begin{vmatrix} \overline{u_1'^2} & \overline{u_1'u_2'} & \overline{u_1'u_3'} & \overline{u_1'\theta'} \\ \overline{u_2'u_1'} & \overline{u_2'^2} & \overline{u_2'u_3'} & \overline{u_2'\theta'} \\ \overline{u_3'u_1'} & \overline{u_3'u_2'} & \overline{u_3'^2} & \overline{u_3'\theta'} \\ \overline{\theta'u_1'} & \overline{\theta'u_2'} & \overline{\theta'u_3'} & \overline{\theta'^2} \end{vmatrix}. \qquad (7)$$

To solve the equations of system (7), it is necessary to know the method of calculating the following values:

$$\overline{u_i'u_j'u_k'}\,;\ \overline{u_i'u_j'\theta'};\ \overline{p'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)};\ \overline{p'\frac{\partial \theta_i'}{\partial x_i}}. \qquad (8)$$

To do this, let's represent quantities (8) in the form of certain linear combinations of the tensor component (8) and the parameter $b^2=\overline{u_k' u_k'}$, which corresponds to the kinetic energy of fluctuations, can be found from the equation (with physical explanations of any term):

$$\frac{\partial b}{\partial t} + \frac{\partial u_k b^2}{\partial x_k} + \frac{\partial}{\partial x_k}\left(\overline{u_k'u_i'u_j'} + 2\overline{u_k'p'}\right) = -2\overline{u_k'u_i'}\frac{\partial u_i}{\partial x_k} - 2\frac{g}{\theta_0}\overline{\omega'\theta'} \qquad (9)$$

Advection	Turbulent diffusion	Effect of forces of tension	Interaction of Reynolds tension & averaged motion	Generation for account of swimming forces

Here g is the magnitude of the acceleration vector due to the planet's gravity, θ_0 is the equilibrium potential temperature; θ', p' are departures from equilibrium values. The equations for the velocity's correlates are in details listed in [7, 8] and Components of tensor of the turbulent tensions are (spectral modes of velocity field):

$$\hat{V}^2 = \sum_{k=1}^{\infty}\sum_{s=-k}^{k} V_{k,s}T_{1,s}^{k}\left(\sum_{q=1}^{\infty}\sum_{j=-q}^{q} V_{q,j}T_{1,j}^{q}\right) = \sum_{k=1}^{\infty}\sum_{s=-k}^{k}\sum_{q=1}^{\infty}\sum_{j=-q}^{q} V_{k,s}V_{q,j} \times \sum_{\nu=|k-q|}^{k+q} \sigma_{1,1,2}^{k,q,\nu}\sigma_{s,j,s+j}^{k,q,\nu}T_{2,s+j}^{\nu} = \overline{v_1'v_1'} = b^2, \qquad (10)$$

Then, according to the well-known closing hypotheses, it is possible to write a system of relevant equations that are usually used for models of the surface layer of the atmosphere:

$$\overline{u_i'u_j'u_k'} = -b\lambda_1\left(\frac{\partial \overline{u_i u_j}}{\partial x_k} + \frac{\partial \overline{u_i u_k}}{\partial x_j} + \frac{\partial \overline{u_j u_k}}{\partial x_i}\right);\ \overline{u_k'u_j'\theta'} = -b\lambda_2\left(\frac{\partial \overline{u_k'\theta'}}{\partial x_j} + \frac{\partial \overline{u_j'\theta'}}{\partial x_k}\right);$$

$$\overline{u_i'\theta'^2} = -b\lambda_3\left(\frac{\partial \overline{\theta'^2}}{\partial x_i}\right);\ \overline{p'\frac{\partial \theta_i'}{\partial x_i}} = -\frac{b}{3l_1}\overline{u_i\theta'} - \frac{1}{3}\delta_{i3}\frac{g}{\theta_0}\overline{\theta'^2};$$

$$\overline{p'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)} = -\frac{b}{3l_1}\left(\overline{u_i u_j} - \frac{1}{3}\delta_{ij}b^2\right) + cb^2\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right); \qquad (11)$$

Here c, l_1, λ_i are constants that specify the scale of turbulent eddies and the degree of their influence on the average motion, as well as the anisotropy of atmospheric turbulence. The theories of closure of systems (6) by relations (11) are universal for all turbulent flows. Specifically for the atmosphere, they are used for the boundary layer, but in a one-dimensional version, namely along the vertical coordinate: $x_3=z$.

For a specific task (determination of turbulence in the thermal "cap" of a concrete industrial city, for example, Odessa or Aleppo or Hamburg or New York etc), it would be correct to abandon the universal closing theories of the components of the tensor (7) and apply the estimation of the energy spectra of its components in the weight fraction of the component b^2, that is, the kinetic energy of turbulent eddies . For example, if $Q_{ij}\left(x_1,x_2,x_3,t\right)$ are the elements of tensor (7), then its reciprocal inverse transformation into an energy structure is [7, 8]:

$$Q_{i,j}\left(x_1,x_2,x_3,t\right)=\int\int\limits_{-\infty}^{\infty}\int E_{i,j}\left(k_1,k_2,k_3,t\right)\exp\left[i\left(k_1x_1+k_2x_2+k_3x_3\right)\right]dk_1dk_2dk_3,$$

$$E_{i,j}\left(k_1,k_2,k_3,t\right)=\int\int\limits_{-\infty}^{\infty}\int Q_{i,j}\left(x_1,x_2,x_3,t\right)\exp\left[-i\left(k_1x_1+k_2x_2+k_3x_3\right)\right]dx_1dx_2dx_3. \quad (12)$$

Next, we apply a comparative energy assessment $E_{i,j}$ for all components Q_{ij}. It is natural that $E_{1,1}=E_{2,2}$, while$E_{3,3}$ significantly differs. With isotropic turbulence in all three directions, the energy estimation process is simplified. Since we are interested in the ventilation of the city in the horizontal direction, we will limit ourselves to a comparative assessment $b=E_{1,1}$.

Further, while developing the theory of turbulent regime in atmosphere of an industrial city, operators of approximation of the energy spectrum are applied with the help of a linear operator of the type (11) and a coupling coefficient (the turbulence coefficient). The components of tensors of the second and third rank describe the processes of nonlinear diffusion and interaction with the mean motion. During diffusion, the process of crushing large vortices into smaller ones is carried out, and when interacting with the average movement, in addition to crushing, there is also a reverse process in nature, during which the size of turbulent vortices stabilizes.

The linear operator is capable of approximating only the linear step part of this process, and equally at all intervals of the spectrum. This is the main drawback of linear closing theories, i.e., in linear closing, only the process of fragmentation (dissipation) takes place over the entire spectrum interval and there is no process of thickening of turbulent vortices due to the merging of energies of small vortices. This clearly contradicts real natural processes, because the laminar flow, passing into the turbulent flow regime, breaks down on the inhomogeneities of the friction layer, which are not directly related to the nature of the turbulence itself. The vortices, entering the free flow mode, should stabilize in size depending on the molecular viscosity of the carrier or on the turbulent viscosity of previously existing turbulent molecules in the medium. It is obvious that the flow, passing through urban buildings, for example, due to collision with it, breaks into a series of vortices that are not balanced with the physical properties of its carrier, and then in free flow it stabilizes in both directions of the spectral interval. Such an effect can be convincingly described within the framework of fractal approaches. The same effect occurs in oncoming traffic streams that merge. This is where the term "turbulent viscosity" becomes clear, which is a pure form of molecular friction.

It is important to note that, since the meaning of the researched process lies in the correct description of the process of stabilization of turbulent vortexes, which are directed differently on separate intervals of the spectrum, then the linear theory naturally only distorts the solution, without introducing useful information into it. According to the linear theory, diffusion from the source is uniformly spread by a spot in isotropic space, while in real diffusion, impurities are captured by large vortices and carried by the flow to much greater distances. This process was called wave or vortex diffusion. It should be noted that this completely clear aspect is still ignored in most works on atmospheric ventilation modelling. Redistribution of energy over the spectrum of eddy sizes is usually called a spectral transformation, the study of which is possible only under the condition of real introduction of nonlinearity into the equation of turbulent motion. In principle, the phenomenon of vortex diffusion must first be described within the framework of an adequate nonlinear theory. This, however, provokes a significant complication of the mathematical apparatus, as in all nonlinear problems.

In the event of a collision of streams with real urban buildings, this transformation process is the main one (and there is no the dissipation of energy into the spectrum of micro-pulsations). Such dissipation was justified in the case of long-term movement of the flow over a substrate surface with uniform roughness (for example, over a forest, sea or field). In the conditions of the city, impurities from the source of pollution can be transferred to much greater distances than during normal diffusion, which introduces ambiguities and creates known problems during the development of recreational activities. As a rule, the application of linear theories of turbulence for the territory of the city is unpromising. This explains why there is still no scientifically based program for the theoretical study of the processes of the spread of harmful impurities in the atmosphere of industrial cities. Moreover, at present, in the conditions of the growth and emergence of new modern megacities, as a rule, the analysis of possible atmospheric ventilation is not carried out, taking into account physical, geographical, climatic, chemical and other factors.

A new model of thermal turbulence spectrum of an industrial city. Let us consider further the effect of thermal turbulence in an industrial city. It is interesting to note that the processes in the thermal "cap" of the city can be determined by analogy with the known soliton of fog formation of the "local" type (e g. [21-23]), which has its own wave and turbulent structure. These structures are tightly connected to each other. Namely, the energy spectra of harmonics of Fourier or Fourier-Bessel transformations can be considered as a spectrum of waves and as a spectrum of turbulent eddies. This is clear from the theories of energy estimates of the spectrum of turbulent pulsations for the urban system. Spectral transformation formulas (12) use the spectral basis of Fourier series, or the Fourier integral. At the same time, the spectral basis of the Fourier-Bessel series for the Fourier-Bessel integral corresponds more closely to the equations of atmospheric dynamics (see, for example, [7,8]).

Given the fact that the spectral-energy function of turbulence is developed only for the Fourier integral, it is more convenient to express it for the Fourier-Bessel basis using, for example, the theory developed in [7,8] in spherical functions, and then use in specific algorithms of the formula of connection of spherical functions with Bessel functions. The searched formula has the following standard form:

$$\lim_{n\to\infty}\left[\frac{1}{n^m}P_n^m\left(1-\frac{z_2}{2n_2}\right)\right]=J_m(z). \qquad (13)$$

In Eq. (13) P_n^m is the adjoined Legendre polynomial, J_m is the Bessel function of the first kind. The most notable property of the tensor vector of spherical functions $T_{m,n}^l$ is that they satisfy the multiplication formula:

$$T_{m,n}^l T_{p,s}^k=\sum_{v=|k-l|}^{k+l}C_{m,p,m+p}^{l,k,v}C_{n,s,n+s}^{l,k,v}T_{m+p,n+s}^v, \qquad (14)$$

Here $C_{m,p,m+p}^{l,k,v}$, $C_{n,s,n+s}^{l,k,v}$ are the Clebsch-Jordan coefficients, which can be calculated by the standard formula:

$$C_{\mathrm{j,k,j}+k}^{l_1,l_2,1}\sqrt{\frac{(2l+1)(l_1+j)!(l-j-k)!(l-l_1+l_2)!(l_1+l_2-l)!(l_1+l_2+l+1)!}{(l_1-j)!(l_2+k)!(l_2-k)!(l+j+k)!(l+l_1-l_2)!}}\times$$
$$\times\sum_{s=\max(j+k,l_1-l_2)}^{l}\frac{(-i)^{l_1+k-s}(l+s)!(l_2+s-j)}{(1-s)!(s-j-k)!(s-l+1+l_2)!(l_1+l_2+s+1)!} \qquad (15)$$

where the lower left index vector-tensor of spherical functions determines the tensor-component number of the set (basis) of these functions for each of the components of the tensor (in our case, the tensor of turbulent stresses). These quantities are well known in quantum mechanics (the useful review is in Refs.[26-28]). By the way, the quantum algorithms (see detailed description in e.g. [26-30]) are useful in solving problems studied here. Let us further introduce the expansion (see, for example, [7, 8]):

$$\widehat{V}=-V_\varphi-iV_\theta=\sum_{l=1}^{\infty}\sum_{n=-l}^{l}V_{l,n}T_{1,n}^l; \qquad \widehat{U}=-U_\varphi-iU_\theta=\sum_{l=1}^{\infty}\sum_{n=-l}^{l}V_{l,n}T_{-1,n}^l;$$
$$V_r=\sum_{l=1}^{\infty}\sum_{n=-l}^{l}W_{l,n}T_{0,n}^l \qquad (16a)$$

where

$$V_{l,n}=v_{l,n}+iv_{l,n};\ U_{l,n}=u_{l,n}+iu_{l,n},\ W_{l,n}=w_{l,n}+iw_{l,n};\ T_{1,n}^l=e^{in\varphi}P_{1,n}^l(\cos\theta);$$
$$T_{-1,n}^l=e^{in\varphi}P_{-1,n}^l(\cos\theta);\ T_{0,n}^l=e^{in\varphi}P_{0,n}^l(\cos\theta). \qquad (16b)$$

The components of turbulent stress tensor are the result of multiplication of series:

$$\widehat{V^2}=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}V_{k,s}T_{l,s}^k\left[\sum_{q=1}^{\infty}\sum_{j=-q}^{\infty}V_{q,j}T_{1,j}^q\right]=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}\sum_{j=-q}^{\infty}\sum_{q=1}^{q}V_{k,s}V_{q,j}\sum_{v=|k-q|}^{k+q}C_{1,1,2}^{k,q,v}C_{s,j,s+j}^{k,q,v}T_{2,s+j}^v,$$

$$\widehat{V}\widehat{U}=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}V_{k,s}T_{l,s}^k\left[\sum_{q=1}^{\infty}\sum_{j=-q}^{\infty}U_{q,j}T_{-1,j}^q\right]=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}\sum_{j=-q}^{\infty}\sum_{q=1}^{q}V_{k,s}U_{q,j}\sum_{v=|k-q|}^{k+q}C_{1,-1,0}^{k,q,v}C_{s,j,s+j}^{k,q,v}T_{0,s+j}^v,$$

$$\widehat{V}V_r=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}V_{k,s}T_{l,s}^k\left[\sum_{q=1}^{\infty}\sum_{j=-q}^{\infty}W_{q,j}T_{0,j}^q\right]=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}\sum_{j=-q}^{\infty}\sum_{q=1}^{q}V_{k,s}W_{q,j}\sum_{v=|k-q|}^{k+q}C_{1,0,1}^{k,q,v}C_{s,j,s+j}^{k,q,v}T_{1,s+j}^v \qquad (17)$$

At the same time onse should write:

$$\widehat{U}\widehat{V}=\widehat{V}\widehat{U}\,;\; V_r\widehat{V}=\widehat{V}V_r\,;\; V_r\widehat{U}=\widehat{U}V_r \qquad (18a)$$

on the basis of the symmetry of the tensor component. In spectral form, this follows from the fact that:

$$\widehat{U}\widehat{V}=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}V_{k,s}T_{-l,s}^{k}\left[\sum_{q=1}^{\infty}\sum_{j=-q}^{\infty}U_{q,j}T_{1,j}^{q}\right]=\sum_{q=1}^{\infty}\sum_{j=-q}^{q}U_{q,j}T_{-l,j}^{q}\left[\sum_{k=1}^{\infty}\sum_{s=-k}^{k}V_{k,s}T_{1,s}^{k}\right],$$

$$\widehat{V^2}=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}V_{k,s}T_{-1,s}^{k}\left[\sum_{q=1}^{\infty}\sum_{j=-q}^{\infty}V_{q,j}T_{-1,j}^{q}\right]=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}\sum_{q=1}^{\infty}\sum_{j=-q}^{q}V_{k,s}V_{q,j}\sum_{v=|k-q|}^{k+q}C_{-1,-1,-2}^{k,q,v}C_{s,j,s+j}^{k,q,v}T_{-2,s+j}^{v},$$

$$\widehat{U}V_r=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}V_{k,s}T_{-l,s}^{k}\left[\sum_{q=1}^{\infty}\sum_{j=-q}^{\infty}W_{q,j}T_{0,j}^{q}\right]=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}\sum_{q=1}^{\infty}\sum_{j=-q}^{q}U_{k,s}W_{q,j}\sum_{v=|k-q|}^{k+q}C_{-1,0,-1}^{k,q,v}C_{s,j,s+j}^{k,q,v}T_{-1,s+j}^{v},$$

$$V_r^2=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}W_{k,s}T_{0,s}^{k}\left[\sum_{q=1}^{\infty}\sum_{j=-q}^{\infty}W_{q,j}T_{0,j}^{q}\right]=\sum_{k=1}^{\infty}\sum_{s=-k}^{k}\sum_{q=1}^{\infty}\sum_{j=-q}^{q}W_{k,s}V_{q,j}\sum_{v=|k-q|}^{k+q}C_{0,0,0}^{k,q,v}C_{s,j,s+j}^{k,q,v}T_{0,s+j}^{v}. \qquad (18b)$$

It is obvious from formulas (18) that the tensor of turbulent stresses decomposes the corresponding components into series by vector-tensor-spherical functions of a certain set indicated by the left subscript:

$$\begin{vmatrix}\widehat{V^2} & \widehat{V}\widehat{U} & \widehat{V}V_r\\ \widehat{U}\widehat{V} & \widehat{U^2} & \widehat{U}V_r\\ V_r\widehat{V} & V_r\widehat{U} & V_r^2\end{vmatrix}=\sum_{l=1}^{\infty}\sum_{n=-l}^{l}\begin{vmatrix}\left[V_{l,n}^{(2)}\right]T_{2,n}^{l} & \left[VU_{l,n}\right]T_{0,n}^{l} & \left[VV_{rl,n}\right]T_{1,n}^{l}\\ \left[VU_{l,n}\right]T_{0,n}^{l} & \left[U_{l,n}^{(2)}\right]T_{-2,n}^{l} & \left[UV_{rl,n}\right]T_{-1,n}^{l}\\ \left[VV_{rl,n}\right]T_{1,n}^{l} & \left[UV_{rl,n}\right]T_{-1,n}^{l} & \left[V_{r\ l,n}^{(2)}\right]T_{0,n}^{l}\end{vmatrix}. \qquad (19)$$

Here, the coefficients of the expansion into the corresponding series of the tensor component are indicated in square brackets. Thus, the components of the turbulent stress tensor are represented linearly, but without the application of "K-theory" (e.g.[7, 8]). The meaning of nonlinearity is reduced to the operation of spectral transformation of energy by wave vector.

Conclusions. Above, we outlined the fundamental, analytical aspects of a new approach to interpreting the process and modelling thermal turbulence in the atmosphere of a standard industrial city. It should be noted that this block of general theory should then naturally be coupled with the theory of turbulence in the atmosphere near urban areas. The principal new moment here is in the further possibility of application the theory of a plane complex field for calculating air circulation in an industrial city's periphery. Within this approach an air flux velocity over a city's periphery in a case of convective instability (the standard situation for the sea industrial city of the Odessa type) can be found by method of plane complex field theory (in analogy with the Karman vortices chain model) [7,24]:

$$v_x-iv_y=\frac{df}{d\varsigma}=\frac{\Gamma}{2\pi i}\left\{\frac{1}{\varsigma-\varsigma_0}+\sum_{k=1}^{\infty}\left(\frac{1}{\varsigma-\varsigma_0-kl}+\frac{1}{\varsigma-\varsigma_0+kl}\right)\right\}+\frac{d}{d\varsigma}\left[\sum_{k=1}^{n}\Gamma_k\ln(\varsigma-b_k)\right]; \qquad (20)$$

Here Γ_k – circulation on the vortex elements, created by clouds, b_k – co-ordinates of these elements, Γ – circulation on the standard Karman chain vortices of, l – distance between standard vortices of the Karman chain, ς – co-ordinate of the convective perturbations line (or front divider) centre, $\varsigma_0 - kl$ – co-ordinate of beginning of the convective perturbation line, $\varsigma_0 + kl$– co-ordinate of end of this line. The indicated parameters are the input model ones and explained in details in Ref. [7, 8].

It is interesting to remind that the processes in the thermal "cap" or heat island zone can be defined by analogy with the known soliton of fogging as a "locale", which has its own wave and turbulent (or chaotic) structure. These structures are rigidly connected to each other. Namely, the energy spectra of harmonics of the Fourier or Fourier-Bessel transforms can be understood both as a wave spectrum and as a spectrum of turbulent vortices (e.g. [7,21]). Specific model applications of the presented approach will be considered in the subsequent works.

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Новий теоретичний підхід до динаміки тепло-масо-переносу, теплової турбулентності і вентиляції повітря в атмосфері промислового міста II. Спектр теплової турбулентності

АНОТАЦІЯ

У даній роботі розробляються фундаментальні аналітичні основи нового послідовного теоретичного підходу до моделювання турбулентного масо-тепло-переносу в атмосфері промислових міст і представлені ключові елементи нової моделі визначення спектру теплової турбулентності промислового міста. Особливу увагу приділено загальним аналітичним аспектам визначення та кількісного урахування достатньо складного феномену хвильової або вихрової дифузії, яке зазвичай ігнорується в більшості сучасних підходів до моделювання атмосферної вентиляції промислових міст. Перерозподіл енергії по спектру вихрових розмірів зазвичай називають спектральним перетворенням, вивчення якого можливе лише за умови реального внесення нелінійності в рівняння турбулентного руху. Представлений підхід імплементується до загальної теорії тепло-масо-обміну, турбулентності та вентиляції повітря в атмосфері промислового міста у комбінації з методом комплексного геофізичного плоского поля та узагальненим підходом Аракави-Шуберта до кількісного опису конвективної нестійкості в атмосфері промислового міста.

Ключові слова: *фізика атмосфери промислового міста, тепломасоперенiс, теплова турбулентність, вентиляція повітря в атмосфері, вихрова дифузія.*