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Modeling of water hammer effect during the single cavitating bubble oscillation

The paper presents the results of an analytical study of the vapor-gas bubble dynamics in cavitation processes, giving consideration to the liquid compressibility. The study is based on the concept that cavitation effects are directly related to the occurrence of hydraulic shock on the surface of an extremely compressed bubble. The purpose of this work is to study cavitation bubble dynamics accounting for the spherical water hammer effects. An equation for the bubble dynamics was obtained, which includes the coefficient of liquid adiabatic compressibility β *as a basic parameter that is directly related to the liquid compressibility. For* $\beta = 0$ *this equation reduces to the classical Rayleigh-Plesset equation for incompressible liquids. The results of a computational experiment performed within the framework of the modified model are presented for which the behavior of a cavitation bubble both in compressible and in incompressible water was analyzed. Based on a detailed analysis of the results obtained, it is shown that over time* $\Delta\tau \propto 1$ μc *.the compressed bubble is in the state of a supercritical fluid with temperature up to 2000 K and pressure of about 400 MPa. The potential energy of the compressed liquid, in the form of a powerful acoustic pulse, emitted by the bubble at the stage of its collapse, is irreversibly dissipated in the surrounding liquid.*

Key words: *hydrodynamic cavitation, bubble dynamics, liquid compressibility, spherical water hammer, acoustic impulse*

Introduction. The problems of cavitation bubble dynamics have attracted the attention of the scientific and industrial community for decades for many applications. To date, it has been established that the effects of cavitation can only be adequately predicted with an allowance for the liquid compressibility. In the existing cavitation models, describing the dynamics of a single bubble, the classical Rayleigh-Plesset equation, which has been derived without taking into account the liquid compressibility, is used as the basic motion equation [1–9].

Despite recent advances in high-speed photography and holography, experimental studies are still unable to provide the necessary information about the final stage of the bubble collapse on a nanosecond scale. The experimental results are limited by the resolution in space and time, especially for micro-bubbles whose size and period are at 10^{-6} m and at 10^{-6}s.

TheoreticalstudiesofcavitationareaimedatdevelopingbubbledynamicsmodelsthatusevariousmodificationsoftheRayleigh–Plessetequation with dueaccount of compressibility effects. Currently, there is a set of approximate equations having the same degree of accuracy and entire equivalent on formal grounds, but with no clear relationship to each other. These equations are used to study the dynamics of single gas bubbles both in the processes of acoustic and hydrodynamic cavitation [1,5,6,8,9]. However, these studies are mainly focused on the behavior of the inner part of the bubble

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while the analysis of the surrounding liquid dynamics during the bubble collapse is very scarce.

At the stage of the bubble compression, liquid moves at high speed towards the bubble center. When liquid suddenly decelerates on the extremely compressed bubblesurfacethe kinetic energy of the liquid ($E_k \propto 10^8\, J$) is transformed into the potential energy of compression with an increase in pressure up to $\Delta p \propto 10^3$ MPa . According to the authors of [6], this situation bears a strong resemblance to the water hammer phenomenon in a duct. As the liquid flow is halted by the abrupt closing of a valve, pressure waves propagate upstream, reflect at the duct inlet, travel downstream to the valve. In the present case the role of the valve is played by the bubble interface, which opposes the inward liquid flow.

The water hammer phenomenon is well known to be an exceptional case in hydraulics, when the liquid compressibility need be accounted for [10]. Therefore, it is interesting to evaluate the possibility of using this phenomenon in modeling the dynamics of cavitation bubbles in a compressible liquid.

This article discusses some features of the gas bubble oscillations in a compressible liquid based on the mathematical model developed in our previous works [4,7]. The focus of this work is to study the physics of compressible cavitation flows and predict the patterns of oscillation and collapse of gas-vapor bubbles, taking into account the spherical water hammer effects.

Formulation of the problem. Consider a spherical bubble with initial radius R_0 incompressible and slightly viscous liquidat pressure p_{l0}, temperature T_{l0} and density ρ_l. The bubble contains saturated vapor at pressure $p_v = p_{sat}(T_{l0})$and non-condensable gas at pressure p_{g0}, so that the total pressure of gas-vapor mixture inside the bubble is $p_{b0} = p_v + p_{g0}$. The equilibrium condition for a bubble with a liquid is determined by the relation [1,7]

$$p_{b0} \equiv p_{g0} + p_v = p_{l0} - 2\sigma(T_{l0})/R_0\,, \tag{1}$$

where $\sigma(T_l)$ is the surface tension. Assuming the gas is ideal and the mass of gas in the bubble m_g is constant, the change in gas pressure inside the bubble during its compression or growth is determined as

$$p_g(\tau) = p_{g0}\left(R_0^3 / R(\tau)^3\right). \tag{2}$$

Starting at $\tau = 0$, liquid pressure away from the bubble $p_{l\infty}$ during a short time $\delta\tau_1$ decreases to the value $p_{l\min} << p_{l0}$ as a result of which the bubble.is activated and then grows under the pressure difference $p_b - p_{l\infty}$ to a maximum size $R_{\max}$ [1,3-5,,7].At instant $\tau_1 = \delta\tau_1$, the liquid pressure $p_{l\infty}$ during a short timeinterval $\delta\tau_2$ increases from $p_{l\min}$ to a final value $p_{sat}(T_l) << p_{fin} \le p_{l0}$. As a result, the bubble is rapidly compressed both under the pressure difference $p_{l\infty} - p_b$ and the sharply increasing capillary pressure $2\sigma(T_l)/R$. The liquid in the vicinity of the bubble moves rapidly in the radial direction towards the bubble center. The pressure value inside the bubble p_b grows due to a great increase in gas pressure $p_g \propto R^{-3}$. When the increasing gas pressure in the bubble becomes equal to the liquid pressure at the bubble wall $p_R(\tau) = p_{l\infty}(\tau) + 2\sigma/R(\tau)$, the velocity of the liquid radial motion at the boundary with the bubble($v_R = dR/d\tau$) will reach its maximum value $v_{R\max}$.

Accordingly, the kinetic energy of the liquid will also be maximum, and the mechanical potential energy of the system is considered to be zero ($E_p = 0$).

Thereafter the liquid moves with deceleration until the final stop, and the bubble reaches its minimum size $R_{\min}$ .With a sudden stop of the liquid on the compressed bubble surface the water hammer effect occurs, as a result of which the liquid kinetic energy is completely converted into the potential energy of the "liquid-bubble" system, and the pressure at the bubble surface p_R reaches its maximum value.

Let us analyze how the kinetic and potential energies of the system change during the compression and subsequent expansion of the bubble, starting from the instant in time, when the potential energy $E_p = 0$. Up to this instant, liquid can be considered as incompressible. A spherical coordinate system is used with the origin at the center of the bubble, which is considered as spherical throughout the process.

Kinetic energy. Let us divide the liquid volume in the vicinity of the bubble into elementary concentric spherical zones of width dr. The volume of the layer at a distance r from the bubblecenter $\delta V(r) = 4\pi r^2 dr$, and the mass of liquid in the layer $\delta m(r) = \text{const}$. The liquid velocity at the boundary with the bubble is $v_R = dR/d\tau$, and the radial motion velocity of the liquid layer at a distance r is $v(r,\tau) = v_R(\tau) R^2/r^2$. Kinetic energy of the liquid inside this layer is

$$\delta E_{kl} = \frac{\delta m \cdot v^2}{2} = \frac{\left(\rho_l \cdot 4\pi r^2 dr\right) \cdot v^2}{2} = \left(2\pi R^4 \rho_l v_R^2\right) \cdot \frac{dr}{r^2}. \tag{3}$$

Integrating the right side of Eq. (3) over the entire volume of the liquid, we find the kinetic energy of the radial motion of the liquid surrounding the bubble

$$E_{kl} = \left(2\pi R^4 \rho_l v_R^2\right) \cdot \int_R^\infty \frac{dr}{r^2} = 2\pi R^3 \rho_l v_R^2 = 3 \cdot \left(\frac{4}{3}\pi R^3 \rho_l \frac{v_R^2}{2}\right) = m_{eff} \frac{v_R^2}{2}. \tag{4}$$

The kinetic energy of an infinite volume of liquid is found be a finite value, equal to $m_{eff} \cdot v_R^2/2$, and mass of this liquid volume. m_{eff} is equivalent to the massof liquid, occupying three times the volume of the bubble.

In a compressible liquid, any change in the kinetic energy in the first layer adjacent to the bubble surface is transferred to the spherical layer at a distance r in time $\Delta\tau_r = (r - R_{\min})/c_{ac}$, where c_{ac} is the speed of sound in the resting liquid. The liquid velocity near the bubble surface is assumed to be $v_R < c_{ac}$ [1-5, 8,9]. When the first liquid layer abruptly stops on the surface of the extremely compressed bubble, i.e. when the spherical water hammer occurs, the kinetic energy of the liquid entire volume is converted into potential energy not instantly, as in an incompressible liquid, but in a finite time. This time can be estimated by determining a minimum distance from the bubble center $r_{\min}$, beyond which the liquid velocity is negligible compared to that near the bubble surface. It follows from the continuity condition that the liquid velocity at a distance r is $v(r) = v_R(\tau)R^2/r^2$.

Assuming that $R_{\min} \propto 1$ мкм and $v_{R\max} \leq 10^3$ m/s which corresponds to the known experimental data [1,5,6,9], we determine the distance $r_{\min}$ beyond which $v(r) \leq 10^{-3}$ m/s. For given conditionsradius $r_{\min} \approx 1$ mm. Therefore, when a spherical water hammer is realized, the transformation of kinetic energy into potential energy in the first layer is transferred to thedistance $r_{\min}$ in time $\Delta\tau_{wh} \approx r_{\min}/c_{ac} \propto 1$ мкс.By analogy with the classical water hammer in pipelines, the distance $r_{\min}$ for the spherical water hammer corresponds to the distance from the shut-off valve to the pipe inlet from a large reservoir [12].

Potential energy. Let us now consider the potential energy change in an elementary layer in the vicinity of the bubble when the liquid decelerates. A decrease in the liquid kinetic energy δE_k in the layeris accompanied by an increase in potential energy δE_p, associated with the work of compressing the layer. In a compressible liquid, any pressure change in the first layer is transmitted sequentially to each layer at sound speed c_{ac}. If the liquid is not compressed,the liquid pressure in the layer $p_r(r)$ is equal to the liquid external pressure $p_{l\infty}$, and the layer initial volume is δV_0.While the layer is compressed, its volume decreases ($\delta V < \delta V_0$). Excess pressure arising in this layer $\Delta p_r = p_r(r) - p_{l\infty}$ is determined as

$$\Delta p = -\frac{\delta V - \delta V_0}{\delta V_0 \cdot \beta}, \tag{5}$$

where β is the coefficient of the liquid adiabatic compressibility.Taking into account Eq.(5), the change in the potential energy of the liquid is defined as

$$dE_{pl} = \int_0^{\delta V - \delta V_0} \Delta p(V) \cdot dV = \frac{(\delta V - \delta V_0)^2}{2\delta V_0 \cdot \beta} = \frac{(\Delta p)^2 \beta \delta V_0}{2}. \tag{6}$$

The change in potential energy in the layer is equal to the kinetic energy change in this layer, which, in accordance with Eq.(3), can be represented as follows

$$dE_{kl} = \frac{dm(\Delta v_r)^2}{2} = \frac{\rho_l \cdot \delta V_0 (\Delta v_r)^2}{2}. \tag{7}$$

Comparing the right-hand sides of Eqs.(6) and (7), we find the relationship between the change in pressure in the layer and the change in velocityin this layer

$$(\Delta p)^2 \beta = \rho (\Delta v_r)^2. \tag{8}$$

The coefficient of adiabatic compressibility is related to the speed of sound by the relation $\beta = \rho c_{ac}^2$ [1]. Substituting this value β into Eq.(8), we arrive atfamousJoukowsky equation, which determines the amount of excess pressure in a liquid during the implementation of the water hammer phenomenon in pipes

$$\Delta p = \sqrt{(\rho/\beta)} \cdot \Delta v_r = \rho c_{ac} \Delta v_r. \tag{9}$$

The change in pressure in a layer at a distance r is determined by the excess pressure $\Delta p_r = p_r(r) - p_{l\infty}$. In the liquid layer adjacent to the bubble surface, the excess pressure is $\Delta p_R = p_R - p_{l\infty}$. Expanding the terms on the right side of Eq.(6), we represent the potential energy of the liquid in the layer at a distance r in the form

$$dE_{pl}(r) = \frac{(\Delta p)^2 \beta \delta V_0 \cdot}{2} = \frac{4\pi (p_r - p_{l\infty})^2 \beta \cdot r^2 dr \cdot}{2}. \tag{10}$$

Using Joukowsky equation both for the first layer and forthe layer at a distance r, we can write, that $(p_R - p_{l\infty}) = \rho c_{ac} v_R$ and $(p_r - p_{l\infty}) = \rho c_{ac} v_r$. This implies

$$p_r - p_{l\infty} = (p_R - p_{l\infty}) \frac{v_r}{v_R} = (p_R - p_{l\infty}) \frac{R^2}{r^2}. \tag{11}$$

Substituting into Eq.(10) the value of $(p_r - p_\infty)$ from Eq.(11), we obtain

$$dE_{pl}(r) = \frac{2\pi (p_R - p_{l\infty})^2 \beta \cdot R^4 dr \cdot}{r^2}. \tag{12}$$

Integrating Eq.(12) within the range from $r = R(\tau)$ to $r \to \infty$, and performing obvious transformations, we find the current value of the potential energy of the entire liquid volume in the process of bubble compression.

$$E_{pl}(\tau) = \frac{4}{3}\pi R^3 \frac{(p_R - p_{l\infty})^2 \beta}{2} \cdot 3 = m_{eff} \frac{(p_R - p_{l\infty})^2}{2} \frac{\beta}{\rho}, \tag{13}$$

where the effective mass m_{eff} has the same physical meaning as in Eq.(4).

Bubble dynamics equationfor compressible liquids. The change in the kinetic energy of the liquid is equal to the sum of the terms that determine the change in the potential energy of the gas in the bubble and of the surrounding liquid.

$$\frac{dE_k}{d\tau} = -\frac{dE_{pl}}{d\tau} - \frac{dE_{pb}}{d\tau}. \tag{14}$$

The values $dE_k/d\tau$ and $dE_{pl}/d\tau$ are found using, respectively, Eqs.(4) and Eqs. (13). The change in the kinetic energy of the liquid per unit time is described as

$$\frac{dE_k}{d\tau} = \frac{d\left(2\pi R^3 \rho_l v_R^2\right)}{d\tau} = 4\pi R^2 \rho \left(\frac{3}{2} v_R^2 + R \frac{dv_R}{d\tau}\right) \cdot \frac{dR}{d\tau}. \tag{15}$$

The change in the potential energy of the liquid per unit time is determined as

$$\frac{dE_{pl}}{d\tau} = \frac{d}{d\tau}\left(2\pi (p_R - p_{l\infty})^2 \beta R^3\right) = 6\pi R^2 (p_R - p_{l\infty})^2 \cdot \beta \cdot \frac{dR}{d\tau}. \tag{16}$$

The change in the potential energy of gas compression in a cavitation bubble per unit time can be described by the equation, which has been presented in [4]

$$\frac{dE_{pb}}{d\tau} = 4\pi R^2 (p_R - p_{l\infty}) \cdot \left(\frac{dR}{d\tau}\right). \tag{17}$$

This equation uses the density and pressure of the gas averaged over the volume of the bubble.Substituting the right-hand sides of Eqs.(15), (16) and (17)into Eq.(14), after carrying out obvious transformations, we arrive an equation that describes the dynamics of a single spherical bubble of a compressible liquid

$$\frac{dv_R}{d\tau}=\frac{p_R-p_\infty+3/2\left(p_R-p_{l\infty}\right)^2\beta-3/2\,\rho v_R^2}{\rho R}. \qquad (18)$$

At $\beta=0$, Eq.(18) reduces to the classical Rayleigh–Plesset equation, which describes the bubbles dynamics in an incompressible liquid

$$\frac{dv_R}{d\tau}=\frac{p_R-p_\infty-3/2\,\rho v_R^2}{\rho R}. \qquad (19)$$

In Eqs.(16)–(19)the liquid pressure at the bubble surface p_R and the gas mixture pressure p_b inside bubble p_b are related by the expression [1–5]

$$p_R=p_b-\frac{2\sigma}{R}-\frac{4\mu_l\cdot v_R}{R}. \qquad (20)$$

Here $\mu_l=f(T_l)$ is the liquid dynamic viscosity at the boundary with the bubble.

Bubbledynamics simulation for compressible liquids. To study the bubble dynamics in cavitation and boiling processes, the previously created unified mathematical model DSB was used, which adequately predicts the behavior of a single vapor-gas bubble in a viscous incompressible liquid with a change in external pressure [4,7,11,13]. Model DSB is applicable in the entire temperature range of the liquid phase existence up to the critical point [4.13].When studying cavitation effects in an incompressible liquid, the Rayleigh-Plesset equation in the form Eq.(19) has used as the motion equation, which with account of liquid compressibilityis replaced by Eq.(18).The system of equations necessarily includes an independent equation for thechange of external pressure in time ($p_{l\infty}=f(\tau)$).

Results and discussion. Using the DSB model, a computational experiment was carried out to study the peculiarities of thegas-vapor bubble oscillation both in compressible and incompressible liquids in the processes of hydrodynamic cavitation.

According to the accepted definition [1,3,4,6], cavitation occurs, if the liquid pressure falls sharply below the saturated vapor pressure ($p_l<p_{sat}(T_l)$), that gives rise to the activation and growth of gas micronuclei, present in the liquid, and then rapidly increases to a value $p_l>p_{sat}(T_l)$, which leads to bubble compression to minimum size. This condition is satisfied in the hydrodynamic cavitation processes, where the drop and increase in pressure is due to the passage of a high-speed liquid flow through a constricting-expanding nozzle, for example, a Venturi tube [1,11,12],

With a high-speed flow through the compression cone and the narrow throat of the Venturi nozzle, the liquid pressure drops quickly to the value $p_l \ll p_{sat}(T_l)$. The subsequent increase in the liquid pressure inside the diffuser leads to oscillation of the extremely compressed cavitation bubble or its irreversible collapse [11,12].

By specifying a suitable nozzle geometry, as well as pressure values at the inlet of the nozzle p_{l0}. at the nozzle throat $p_{\min}$ and outlet of the nozzle. p_{fin}, one can calculate,with using the Bernoulli equation,the change in pressure $p_l=f(\tau)$ in a fixed liquidelement as it flows through the nozzle [11].

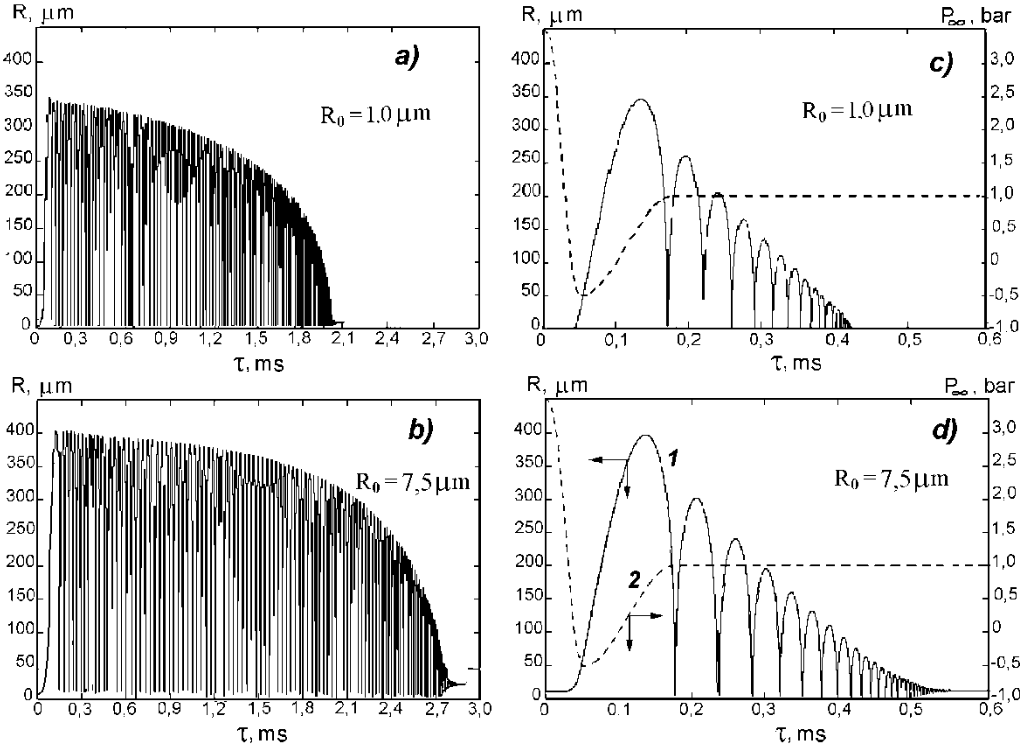

Fig.1. The change in the bubble radius with time during bubble oscillations in waterwith accounting for the liquid incompressibility (***a, b***) and compressibility (***c, d***) for two initial sizes of gas-vapor nuclei: R_0=1 µm (***a, c***) and R_0=7.5 µm (***b, d***). The dotted lines in figures (c, d) show the change in the liquid external pressure $p_{l\infty} = f(\tau)$. Calculation according to the DSB model under the conditions**:** p_{l0}=3.5 bar; p_{fin} =1 bar; T_{l0}=393 K

The computational experiment was carried out for water with temperature $T_{l0} = 293$ K at the constant pressurevalues**:** p_{l0}=3.5 bar, $p_{\min}$ =–0.5 bar and p_{fin}=1 bar. The time intervals $\delta\tau_1$=0.5 ms and $\delta\tau_2$=1.25 ms, which determine the duration of the pressure change from p_{l0} to $p_{\min}$ and from $p_{\min}$ to p_{fin}, respectively, also kept constant. The evolution of single vapor-gas nuclei with initial radii R_0= 1 µm, 2.5 µm, 5 µm, and 7.5 µm was considered.

Figure 1 shows the comparative characteristics of the bubble oscillations calculated inboth the incompressible and the compressible water for two activated gas-vapor nuclei with initial sizes R_0=1µm and R_0=7,5 µm.

The calculation was carried out by using the Rayleigh Plesset equation in the form of Eq.(19) for the incompressible liquid, and using Eq. (18) for the compressible that.The change in the external pressure $p_{l\infty} = f(\tau)$, which determines the activation, growth and subsequent compression of the bubbles in both the compressible and incompressible water, is shown in Figs 1-c and 1-d by dashed lines.

The data presented in Fig.1.confirm the previously established regularity, that in typical cavitating flows the maximum bubble size is about 100 times the initial size of the gaseous nucleus [1–3].The figures show also that in an incompressible liquid the duration of bubble oscillations until the final collapse is almost five times longer than in a compressible liquid.

This is explained by the fact that, neglecting the liquid compressibility, the energy dissipation of in an oscillating bubble is due to the influence of interfacial heat and mass transfer, the effects of the liquid viscosity at the boundary with the bubble, and the degree of compression of the bubble. [4,7]. All these factors were taken into account in the DEP model.

Brennen [1], referring to the work of Chapman and Plesset [2], indicates that the damping of bubble oscillations is directly related to liquid viscosity, the liquid compressibility through acoustic radiation, and is also due to thermal conductivity. These three damping components are conveniently represented as three additive contributions to the effective viscosity μ_{ef}**:** respectively, the actual liquid viscosity μ_l, "acoustic" μ_{ac} and "thermal" μ_T viscosities , which can then be used in the Rayleigh-Plesset equation in the form $\mu_{ef} = \mu_l + \mu_{ef} + \mu_{ef}$ instead of the actual fluid viscosity μ_l, The calculated data show that the components μ_l and μ_T are predominant, rather than the compressibility factor μ_{ac} [1,2]. In the absence of dissipation mechanisms such as viscosity, the oscillations would continue indefinitely without damping [1]. A similar approach with the introduction of various corrections for the effective viscous pressure is also used in other bubble dynamics models to study the mechanism of oscillation damping in compressible liquids [5,6,8,9].

The data presented in Fig. 1 demonstrate that the damping mechanism of bubble oscillations in compressible liquids can be explained within the water hammer concept without introducing any fitting corrections, if the model includes physical factors responsible for the bubble behavior at the collapse stage.

In an incompressible liquid (Fig. 1 a, b), long-term bubble oscillation with a slow decrease in amplitude is conditioned by the minor energy losseswhich aredue to the liquidviscosity μ_l, thermal effects and the kinetics of phase transitions.

In a compressible liquid, in each oscillation cycle, the energy losses are primarily associated with the conversion of the potential energy of the compressed water at $R = R_{\min}$ into a power acoustic pulse, which is irreversibly dissipated in the surrounding liquid. As a result, the compressed gas bubble cannot recover to its previous size $R_{\max}$ due to a significant energy loss, which isshown in Fig.1c,d.

Figure 2 demonstrates the fact, that in each cycle of oscillations during the liquid compression and subsequent stretching near the bubble surface , the bubble radius value $R_{\min}$ remains constant and physical parameters of the gas mixture inside the bubble (T_b, p_b), reaching their maximum values, also remain unchanged. The duration of the spherical water hammer ($2\Delta\tau_{wh}$), i.e. the residence time of the compressed bubble at rest, depends on the value $R_{\min}$ in a given oscillation cycle.

An analysis of the obtained results shows that the main significance of the liquid compressibility lies not so much in its relatively weak effect on the bubble dynamics, but in the role that it plays in the formation of the water hammer acoustic pulses during the bubble recovery following its collapse. With an increase in gas amount in the bubble $m_g = f(R_0, p_{b0})$, the efficiency of the water hammer action decreases.The liquid compressibility is known to damp the bubble oscillation amplitude in each os-

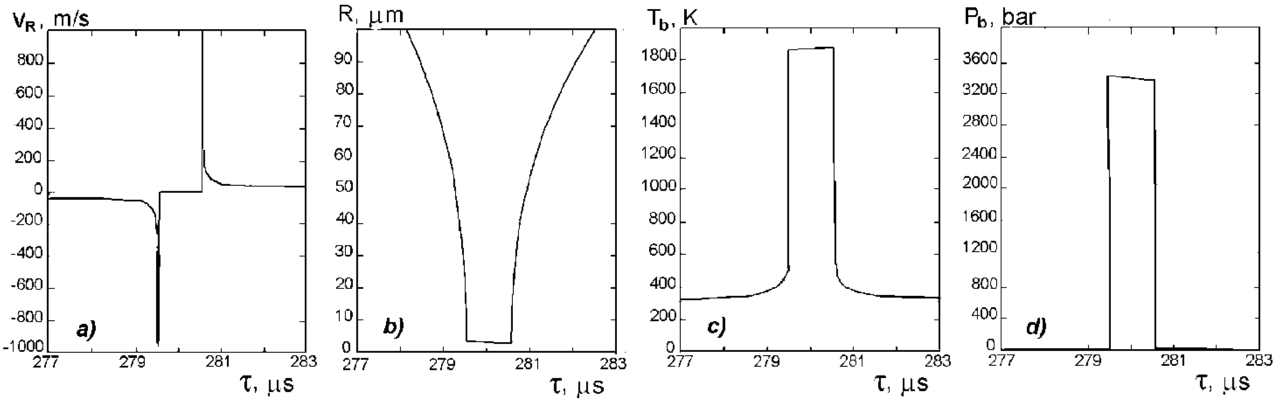

Fig.2. The change in the liquid velocity at the boundary with the bubble $v_R(\tau)$ (***a***); the bubble radius. $R(\tau)$ (***b***); temperature. $T_b(\tau)$ (***c***) and pressure $p_b(\tau)$ (***d***) of the gas-vapor mixture inside the bubble at the stage of its compression in one of the period of its oscillation in a compressible water.
Calculation according to the DEP model**:** R_0=2.5 μm; p_{l0}=3.5 bar; p_{fin}=1 bar; T_{l0}=393 K.

cillation cycle, but it is still unclear to what extent this effect is accurately captured by weakly compressible versions of the Rayleigh-Plesset equation [1, 6].

In this study, the calculated maximum values of gas pressure inside the bubble at the stage of its collapse are $p_b \le 400$ MPa, and the maximum values of temperature are $T_b \le 2000$ K.According to known experimental and calculated data [1, 4–7], pressure values p_b inside extremely compressed bubbles can reach 10^3 MPa, and temperature values T_b can exceed 10^4 K, that is much higher than the critical values of these parameters, which for water are 22.5 MPa and 647 K, respectively.

In such situation, the conception of an interfacial surface loses its physical meaning. During $\Delta\tau \propto 1\mu s$ the substance inside a spherical micro-volume will be in a state of supercritical fluid (SCF) (neither liquid nor vapor). In this local zone with diameter $d \propto 10$ μm an anomalously high temperature gradient $\nabla T \approx 10^8$ K/m appears [4,7,13]. These phenomena evidently associated with interfacial instability, that canlead to the cavitation bubbledestruction and its fragmentation into many small micro-bubbles, which is recorded in experiments[1,9]. These effects were analyzed in detail in [13] without accounting for the liquid compressibility and, to a certain extent, were used in the equations of the DSB model. This allows a more correct description of the behavior of an extremely compressed bubble, since none of the known bubble dynamics models gives consideration to possibility of a substance transition in the “bubble-liquid” system to the supercritical region

Most bubble dynamics models deal mainly with spherical bubbles on the assumption that the spherical shape is stable during bubble expansion, but it is not stable during bubble compression, which, according to the authors, explains the reason for the bubble destruction. It is believed that the bubble destruction and fragmentation into small micro-bubbles, excludes the possibility of emission of acoustic shock pulses into the liquid volume [3,6]. In addition, a correct theoretical study cannot be carried out also because the phase diagram of water for supercritical values of temperature and pressure is currently not well known.

Conclusion This analytical study was carried out within the generally accepted assumptions about the existence of a liquid-gas interface at all stages of the evolution of a spherical cavitation bubble, including the collapse and recovery stages. Obviously, in terms of further research, it is of interest to consider the development of hydraulic shock on the surface of a collapsing bubble under the conditions of the short-term disappearance of the interface and the transition of a substance to the state of a supercritical fluid.

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Моделювання ефекту гідравлічного удару при осциляції одиночної кавітаційної бульбашки

АНОТАЦІЯ

Дослідження виконувалося з метою модифікації моделі динаміки парогазової бульбашки з урахуванням стисливості рідини, що істотно впливає на точність оцінки енергетичних кавітаційних впливів. Теоретичні дослідження кавітаційних процесів спрямовані на розробку моделей динаміки бульбашок, в яких для урахування фактора стисливості застосовуються різні модифікації класичного рівняння Релея - Плесета для нестисливих рідин. Ці дослідження зосереджені на поведінці газової фази всередині бульбашки, тоді процеси в рідині в околиці бульбашки вивчено недостатньо. Вочевидь, що природа кавітаційних ефектів прямо пов'язана із виникненням гідравлічного удару при миттєвій зупинці радіальної течії рідини на поверхні гранично стиснутої бульбашки. В такій постановці динаміка кавітаційних бульбашок досі не розглядалася. Метою даної роботи є дослідження динаміки кавітаційної бульбашок з урахуванням дії сферичного гідроудару. В плані поставленої задачі одержано рівняння динаміки бульбашки, в якому, як параметр, що враховує стисливість, застосовується коефіцієнт адіабатичної стисливості рідини β, *пов'язаний зі швидкістю звуку в рідині* c_{ac} *співвідношенням* $\beta = \rho c_{ac}^2$. *При* $\beta = 0$ *рівнянні зводиться до класичного рівняння Релея - Плесета, Наведено результати обчислювального експерименту, в якому досліджено поведінку кавітаційної бульбашки як в стисливій так і в нестисливій воді. Показано, що протягом короткого часу* $\Delta\tau \propto 1$ мкс *стиснена бульбашка перебуває в стані надкритичного флюїду з температурою до 2000 К і тиском близько 400 МПа, Потенційна енергія стиснутої рідини, у вигляді потужного акустичного імпульсу, що випромінюється на стадії колапсу, безповоротно дисипується в навколишній рідині.*

Ключові слова: *гідродинамічна кавітація, динаміка бульбашок, стисливість рідини, сферичний гідравлічний удар, акустичний імпульс.*