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Negative ion formation under stationary working mode of cylindrical ionic loudspeaker:anode weak isolation limit

In this paper we consider stationary mode of the ionic loudspeaker with cylindrical geometry. The formulas for the total number and number density of negative ions created are obtained. It is assumed that the leakage of the electric charge is due to weak isolation of the anode. The estimate for the acoustic pressure level is given.

Introduction.

The majority of the modern acoustic systems and other devices used for the sound generation suffer from a number of disadvantages. Among them are high cost of high-quality generators and low efficiency. The later factor limits the size of the device with the growth of the acoustic power. In particular, these disadvantages are inherent to common loudspeakers [1].

Such drawbacks are absent at least partly in "ionic loudspeaker", which has been constructed by M.V. Chizhov. In this paper we consider the configuration of cylindrical geometry for this sound generator. Such configuration happens to be very effective for the sound generation (see Fig. 1).

The anode is a cylindrical shape conducting grid with the radius R covered with some material of high resistivity. We will consider the weak isolation condition for the anode. In such a case the accumulation of the charges occurs close to its surface. Thus a grid cell size is big enough for charges penetrate almost freely with subsequent recombination. Also the width of the covering layer should be small. We ne-

Fig. 1. *Ionic loudspeaker of cylindrical geometry (side and front views).*

glect the influence of the anode surface roughness. This allows to neglect additional corona effects which produce the positive ions.

The cathode is a set of conducting needles placed regularly along vertical axis with small distance between them. In calculations we substitute it by the coaxial (with respect to anode) cylinder of small radius $a << R$. Such device operates in the following way. The potentials 0 and $U > 0$ are applied to cathode and anode correspondingly. The polarization of the nitrogen N_2 and the oxygen O_2 molecules in the air occurs. They become polarized with the corresponding induced dipole moment and additional potential energy due to applied electric field. Due to electric field non-uniformity these molecules are subjected to the force directed towards the cathode (needles). The oxygen molecule captures the electron on the surface due to a high affinity [2, 3]. As a result near the cathode the negative ions O_2^- are produced. This is one of the typical mechanism charge leakage from the sharp end. Other reactions can be ignored for simplification.

If the applied voltage U is constant then the corresponding electric current occurs. Reaching the electrode surface the ions either release their charges or leave the system which in general is not closed. If the alternating voltage is applied additionally, then the oscillating motion occurs. These oscillations are transferred to the neutral molecules N_2 and O_2 due to collisions. The molecules collective synchronized motion generates the sound.

This study is aimed at calculating the number density of the negative ion within the loudspeaker under stationary operation mode ($U = const$). At first we calculate the potential and the strength of the corresponding electric field without charge leakage account. Then we take into consideration this effect as the first order smallness. As a result we obtain the formulas for the number density and the total number of the negative ions. The acoustic pressure is estimated too.

Electric field in the absence of charge leakage.

We begin with the calculation of the potential φ and the electric field $\mathbf{E}$ distributions neglecting the charge leakage (zeroth order approximation). For the geometry considered these distributions are well known (see e.g. [4]) and are given by:

$$\varphi = U\frac{\ln\frac{r}{a}}{\ln\frac{R}{a}}, \quad E_r = -\frac{d\varphi}{dr} = -\frac{U}{r\ln\frac{R}{a}} = -E < 0, \tag{1}$$

where E_r is the radial component of the electric field $\mathbf{E}$ and E its absolute value.:

$$E = |\mathbf{E}| = |E_r| = \frac{U}{r\ln\frac{R}{a}}. \tag{2}$$

The potential φ (1) obeys the Laplace equation

$$\Delta\varphi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\psi^2} + \frac{\partial^2\varphi}{\partial z^2} = \frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi}{dr}\right) = 0, \tag{3}$$

where the cylindrical coordinates (r, ψ, z) are used and the corresponding symmetry is taken into account. The boundary conditions are as follows:

$$\varphi(a) = 0, \quad \varphi(R) = U. \tag{4}$$

Note that we consider on the region between electrodes.

The electric field strength at the cathode $r = a$ according to (2) is

$$E_{surface} = \frac{U}{a \ln \frac{R}{a}}. \quad 5)$$

The value $E_{surface}$ can be very high if the radius a is small enough and therefore the charge leakage should be taken into account in this case.

Note that expressions (1), (2), (5) are valid also for the alternating voltage U which smoothly varies with time t. For example, they are applicable for the sound frequencies.

The account of charge leakage in the first order approximation.

Let's take into consideration the charge leakage in the stationary mode. We designate the number density of the negative O_2^- ions as n_i. The continuity equation is:

$$\frac{\partial n_i}{\partial t} - k_i \nabla (n_i \nabla \varphi) = \frac{\partial n_i}{\partial t} - k_i \frac{1}{r} \frac{\partial}{\partial r} \left(n_i r \frac{\partial \varphi}{\partial r} \right) = 0, \tag{6}$$

where k_i stands for the ions mobility and is about $1.4 \cdot 10^{-4}$ $m^2/(V \cdot s)$. The equation (6) is obtained by using the standard continuity equation

$$\frac{\partial n_i}{\partial t} + \nabla \mathbf{j}_i = 0. \tag{7}$$

with account common definitions $\mathbf{j}_i = n_i \mathbf{v}_i$ и $\mathbf{v}_i = k_i \mathbf{E} = -k_i \nabla \varphi$ for the current density and the mobility k_i.

In the zeroth order approximation the ions are absent and the potential φ and the strength E are given by (1), (2), we can derive from (6) in the first order on n_i

$$\frac{\partial n_i}{\partial t} - \frac{k_i U}{\ln \frac{R}{a}} \frac{1}{r} \frac{\partial n_i}{\partial r} = 0, \tag{8}$$

In stationary mode $\frac{\partial n_i}{\partial t} = 0$ and from (8) we get

$$\frac{\partial n_i}{\partial r} = 0, \quad n_i = \text{const}, \tag{9}$$

i.e. the number density is constant. In order to determine its value we need to set the corresponding boundary condition at $r = a$. For this we should analyze the charge leakage mechanism. The additional potential energy of the oxygen molecule equals $W = -\varepsilon_0 \alpha E^2 / 2$, where $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m^{-1} is the electrostatic constant α is the polarizability which is about $1.56 \cdot 10^{-30}$ m^3. The corresponding radial force is

$$F_r = -\frac{dW}{dr} = \varepsilon_0 \alpha E \frac{dE}{dr} = -\frac{\varepsilon_0 \alpha U^2}{r^3 \ln^2 \frac{R}{a}} < 0, \qquad (10)$$

here we use the first order approximation and the expression (2). The radial velocity of the molecule (the mass $m = 5.31 \cdot 10^{-26}$ kg) can be estimated based on the energy conservation law:

$$\frac{mv_r^2}{2} + W = \frac{mv_r^2}{2} - \frac{\varepsilon_0 \alpha E^2}{2} = \text{const}. \qquad (11)$$

We neglect the changes in the partial densities and the pressures of nitrogen N_2 and oxygen O_2 under the field influence. Therefore the density n of the oxygen molecules O_2 is assumed to be constant with the value $5.25 \cdot 10^{24}$ m^{-3}. The constant in the right hand side of (11) can be determined from the condition $rv_r = \text{const}$, derived from the continuity equation. It equals to zero. By substitution (2) into (11), with account of (1) results we get

$$v_r = -\sqrt{\frac{\varepsilon_0 \alpha}{m}} E = \sqrt{\frac{\varepsilon_0 \alpha}{m}} E_r = -\sqrt{\frac{\varepsilon_0 \alpha}{m}} \frac{U}{r \ln \frac{R}{a}} < 0, \quad v_r\big|_{r=a} = -\sqrt{\frac{\varepsilon_0 \alpha}{m}} \frac{U}{a \ln \frac{R}{a}}. \qquad (12)$$

Note that in SI units the factor $\sqrt{\varepsilon_0 \alpha / m}$ in (12) formally can be considered as the mobility for the polarized molecules and it is approximately $1.61 \cdot 10^{-8}$ m^2/(V·s).

On the cathode surface ($r = a$) the absolute value of the current density of the negative ions O_2^- is proportional to the value of neutral molecules O_2 density. Yet it is proportional to the total charge accumulated by cathode. The latter value is proportional to $S_{surface} E_{surface}$, where $S_{surface} = 2\pi a l$ – is the total area of the inner cylinder, l is its height. Therefore we can write up to some factor:

$$n_i k_i E_{surface} \sim n(-v_r)\big|_{r=a} S_{surface} E_{surface}. \qquad (13)$$

Note that (13) can be written more appropriate by introduction of some Boltzman type factor which accounts for the probability of the electronic output with subsequent attachment to the molecule O_2. Substituting (5) and (12) into (13) up to some factor we obtain

$$n_i \sim S_{surface} E_{surface} \sim \frac{lU}{\ln \frac{R}{a}}. \qquad (14)$$

To get more exact expression one can change $S_{surface}$ by some effective area $S_{effective}$ for all emitters (needles). For the system shown in Fig. 1, we use the following values: $R = 1.82$ cm, $a = 0.02$ cm, $l = 24$ cm, $S_{effective} \approx \pi a^2 \cdot 230 \approx 0.29$ cm^2 (where 230 – is the number of needles).

Therefore $S_{effective} / (2\pi a l) \approx a \cdot 230 / (2l) \approx 0.10$. Finally instead of (14) we will use the following expression

$$n_i \sim S_{effective} E_{surface} = \frac{S_{effective} U}{a \ln \frac{R}{a}}. \qquad (15)$$

Thus in stationary mode in the first approximation of ions number their density is constant and given by (15). The total number of O_2^- ions is

$$N_i = \pi \left(R^2 - a^2 \right) l n_i \sim \frac{\left(R^2 - a^2 \right) l S_{effective} U}{a \ln \frac{R}{a}}. \qquad (16)$$

Note that (16) is valid for the coaxial cylindrical electrodes. In a case they are not full cylinders (as shown in Fig. 1) then the additional factor $\psi_0 / (2\pi)$, appears where ψ_0 is the corresponding angle (for the system considered $\psi_0 = \pi$). So instead of (16) we will use the formula

$$N_i \sim \frac{\psi_0 \left(R^2 - a^2 \right) l S_{effective} U}{a \ln \frac{R}{a}}. \qquad (17)$$

To estimate the acoustic pressure p under alternating field with the amplitude E_0 it is sufficient the value $N_i e E_0$ (total force) divide by the value $\psi_0 R l$ (surface area). Taking into account (17) we obtain

$$p = \frac{N_i e E_0}{\psi_0 R l} \sim \frac{\left(R^2 - a^2 \right) S_{effective} U E_0}{R a \ln \frac{R}{a}}. \qquad (18)$$

Corresponding sound pressure level (SPL) in decibels is

$$SPL = 20 \lg \frac{p}{p_0} = 20 \lg \left[\frac{\left(R^2 - a^2 \right) S_{effective} U E_0}{p_0 R a \ln \frac{R}{a}} \right] + \text{const}, \qquad (19)$$

where $p_0 = 2 \cdot 10^{-5}$ Pa. The determination of factors in expressions (13)-(18) and therefore the constant in (19) is complicated. To estimate the number density n_i and total number N_i of the negative ions and the SPL it is sufficient to use the volt-ampere characteristic (VAC) which is shown in Fig. 2.

For current values shown in VAC the charge density and the total number of ions are $n_i \sim 10^{15 \div 16}$ m^{-3} and $N_i \sim 10^{11 \div 12}$ correspondingly. If we take the amplitude $E_0 \sim 10^{4 \div 5}$ V/m of the alternating signal we obtain $SPL \sim 70$ decibels.

Exact calculation of a charge leakage.

In this section we describe how to account the charge leakage exactly. We start with equation (6) which in stationary mode gives

$$n_i r \frac{d\varphi}{dr} = C_1, \qquad (20)$$

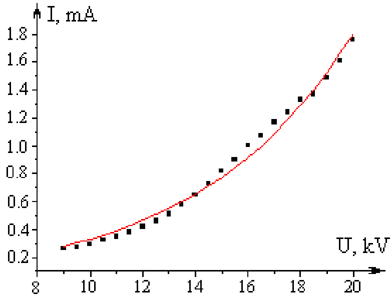

Fig. 2. *VAC for the loudspeaker of cylindrical geometry*

where $C_1 > 0$ is the integration constant. Instead of the Laplace equation (3) the Poisson equation should be used

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi}{dr}\right) = -\frac{e}{\varepsilon_0}n_i . \tag{21}$$

From (20) the combination $r\frac{d\varphi}{dr}$ is determined and its substitution into (21) yields

$$\frac{1}{r}\frac{d}{dr}\left(\frac{C_1}{n_i}\right) = -\frac{e}{\varepsilon_0}n_i , \quad \frac{C_1}{n_i^3}\frac{dn_i}{dr} = \frac{e}{\varepsilon_0}r , \quad n_i = \sqrt{\frac{C_1}{C_2 - \frac{e}{\varepsilon_0}r^2}} , \tag{22}$$

where $C_2 > 0$ is the second integration constant. Substituting (22) into (20) we obtain

$$\frac{d\varphi}{dr} = \frac{1}{r}\sqrt{C_1\left(C_2 - \frac{e}{\varepsilon_0}r^2\right)} ,$$

$$\varphi = \sqrt{C_1\left(C_2 - \frac{e}{\varepsilon_0}r^2\right)} - \sqrt{C_1C_2}\ln\left(\frac{\sqrt{C_2} + \sqrt{C_2 - \frac{e}{\varepsilon_0}r^2}}{\sqrt{\frac{e}{\varepsilon_0}}r}\right) + C_3 , \tag{23}$$

where C_3 is the third integration constant. The constants C_1, C_2, C_3 can be found from boundary conditions (4) and the relation $n_i \sim S_{surface}E_{surface}$..

Discussion.

We obtain approximate expressions (15) and (17) for the density and the total number of the negative ions in ionic sound generator of cylindrical geometry in stationary mode. We obtain the corresponding sound pressure level (19) and estimate its value.

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Чижов М.В., Маренков В.И., Эйнгорн М.В.

Образование отрицательных ионов при стационарном режиме работы цилиндрического ионного громкоговорителя: предельный случай проницаемой изоляции анода

АННОТАЦИЯ

В данной статье получены формулы для плотности и полного количества отрицательных ионов, которые образуются в ионном громкоговорителе цилиндрической формы при стационарном режиме его работы. Предполагается, что стекание электрического заряда происходит при слабой изоляции анода. Произведена численная оценка соответствующего уровня звукового давления.

Чижов М.В., Маренков В.І., Ейнгорн М.В.

Утворення негативних іонів при стаціонарному режимі роботи циліндричного іонного гучномовця: граничний випадок проникаючої ізоляції анода.

АНОТАЦІЯ

У даній статті отримано формули для густини та інтегральної кількості негативних іонів, які утворюються в іонному гучномовці циліндричної форми при стаціонарному режимі його роботи. Передбачається, що стікання електричного заряду відбувається при слабкій ізоляції анода. Здійснена чисельна оцінка відповідного рівня звукового тиску.